Heavy quark mass expansion and intrinsic charm in light hadrons

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Abstract

We review the technique of heavy quark mass expansion of various operators made of heavy quark fields using a semiclassical approximation. It corresponds to an operator product expansion in the form of series in the inverse heavy quark mass. This technique applied recently to the axial current is used to estimate the charm content of the η, $\eta\prime$ mesons and the intrinsic charm contribution to the proton spin. The derivation of heavy quark mass expansion for $\langle \bar{Q}\gamma_5 Q\rangle$ is given here in detail and the expansions of the scalar, vector and tensor current and of $\langle \bar{Q}\nabla_\mu \gamma_\nu Q\rangle$ (a contribution to the energy-momentum tensor) are presented as well. The obtained results are used to estimate the intrinsic charm contribution to various observables.

1 Introduction

Nowdays it is established beyond any doubts that the naive picture of light hadrons as made of three constituent quarks (for baryons) or $q\bar{q}$ pairs of constituent quarks (for mesons) is not complete. The DIS experiments revealed the rich sea structure of the nucleon, these experiments showed in particular that a considerable portion of the nucleon spin is carried by the strange component of the nucleon sea. Furthemore there are experimental facts which seems to suggest that a non-vanishing nonperturbative component of intrinsic charm is present in light hadrons [1, 2].

We address the problem of intrinsic charm content of light hadrons from the point of view of the heavy quark mass expansion. The $c\bar{c}$ pairs in light hadrons, due to parametrically large mass of charm quarks, can appear in a light hadron as virtual state whose life time is short, of order $1/m_c$. The nonperturbative (with typical momenta below heavy quark mass m_c) gluon and light quark fluctuations are slowly varying from "point view" of virtual $c\bar{c}$ pair, hence the heavy quark mass expansion is equivalent to the semiclassical expansion. This expansion allows to rewrite operators made of heavy quarks in terms of light degrees of freedom (gluons and light quarks). For a detailed discussion of the heavy quark mass expansion see ref. [3].

Let us note also that in absence of a direct probe of gluons the open charm production is considered as the main source of information about nucleon's gluon distributions. In hard leptoproduction heavy quarks are produced in the leading order via the photon-gluon fusion (PGF). The leading graph for PGF can be related directly to gluon distributions

if one assumes that there is no intrinsic charm content in the nucleon (no $c(x), \bar{c}(x)$ and no $\Delta c(x), \Delta \bar{c}(x)$ at normalzation point $\mu = m_c$). However now there are many evidences that, in principle, there might be considerable intrinsic charm component in the nucleon wave function even at low normalization point. For reliable extraction of gluon distributions from open charm electroproduction experiments it is necessary to have quantitative estimates of the intrinsic charm content of the nucleon.

This paper will be organized as follows: In the first part we present the calculation of the expectation value of heavy quark currents in the background of gluon fields using a semiclassical approximation. This corresponds to an expansion in the inverse of the heavy quark mass

$$\langle Q^{\dagger}(x)\Gamma Q(x)\rangle = \sum_n \frac{1}{m^n} X_{\Gamma}^{(n)} \, , \tag{1}$$

where Γ denotes the Lorentz structure of the current and the $X_{\Gamma}^{(n)}$ are local expressions of the field strength depending on Γ. In section 2.1 we review the large m expansion of the fermion determinant appearing in our definition of the expectation value. In section 2.2 we then outline the expansion of color singlet currents in general before we present in section 2.2.1 the expansion of the axial current used in [4] in full detail. The connection to the expectation value of the axial vector current using the axial anomaly equation and some general restriction coming from this equation are given in section 2.2.2. As further examples we present the expansion of the scalar current in section 2.2.3, the vector current (section 2.2.4) and the tensor current (section 2.2.5), respectively. In section 2.2.6 we finally show the result of the expansion of $\langle Q^{\dagger}(x)\nabla_{\mu}\gamma_{\nu}Q(x)\rangle$, appearing in the energy-momentum tensor of QCD.

In the second part we discuss the calculation of intrinsic heavy quark content of light hadrons as an application of the heavy quark mass expansion. In the case of charm content of η', η mesons and intrinsic charm contributions to the proton spin we reduce the calculations of these quantities to matrix elements which are already known either phenomenologically or were computed previously. In other cases, like intrinsic charm contribution to the nucleon tensor charge and to energy momentum tensor, the problem is reduced to matrix elements of gluon operators which can be estimated using various nonperturbative methods in QCD: lattice calculation, QCD sum rule, theory of instanton vacuum.

2 Heavy quark expansion of currents in the background gluon and light quark fields

The expectation value of a color-singlet quark current made of heavy quarks in the background of gluon and light quark fields can be written after integration out heavy degrees of freedom as:

$$\langle Q^{\dagger}(x)\Gamma Q(x)\rangle = \det D \, \operatorname{tr}_{c,\gamma}\langle x|\frac{1}{D}\Gamma|x\rangle \, , \tag{2}$$

Here Γ denotes an arbitrary Lorentz-structure. Note that all calculations will be performed in the euclidean space-time, so the QCD Dirac operator reads:

$$D = i\slashed{\nabla} + im \qquad (3)$$

where the covariant derivative is defined as

$$\nabla_\mu = \left(\partial_\mu - i\frac{\lambda^a}{2} A_\mu^a(x) \right) \qquad (4)$$

and m is the heavy quark mass. For the used conventions and the euclidization see the appendix. Eq.(2) can now be expanded in a power-series of the inverse heavy quark mass, $1/m$, under the assumption that the gradient of the background field strength is small compared to m. The expansion of determinant of the Dirac operator in eq.(2) has been calculated by a large number of authors, see e.g. [5, 6, 7]. We briefly review the calculation of the determinant following [7] since we use the result of this calculation as a check of the expansion of a scalar current of heavy quarks in Section 2.2.3.

2.1 Expansion of the determinant

The expansion of the determinant for heavy quarks in eq.(2) yields divergences of various types. Since most of these divergences are connected with the determinant of the free Dirac operator we normalize the determinant with that in zero external field. For the remaining infinity which can be related to the logarithmic renormalization of the coupling constant, we use the so-called ζ-regularization. Using the identity

$$\det D = \det D^\dagger = \left(\det D^\dagger D \right)^{\frac{1}{2}} \qquad (5)$$

the normalized and regularized determinant can be written as follows:

$$\left(\frac{\det D}{\det D_0} \right)_{\zeta-\mathrm{reg}} = \exp\left[-\frac{1}{2}\lim_{s\to 0} \frac{\mathrm{d}}{\mathrm{d}s} \frac{M^{2s}}{\Gamma(s)} \int_0^\infty \mathrm{d}t\, t^{s-1}\, \mathrm{Tr}\left[e^{-tD^\dagger D} - e^{-tD_0^\dagger D_0} \right] \right], \qquad (6)$$

where D_0 denotes the Dirac operator in the absence of external gluon fields and M is the regulator mass. The functional trace denoted by Tr in eq.(6) can be calculated with respect to any complete set of states. For further calculations it is convinient to compute functional traces in the basis of plane waves, so that

$$
\begin{aligned}
\mathrm{Tr}\left[e^{-tD^\dagger D} \right] &= \mathrm{tr}_{c,\gamma} \int \mathrm{d}^4x \int \frac{\mathrm{d}^4 k}{(2\pi)^4} e^{-ikx} \left[e^{-tD^\dagger D} \right] e^{ikx} \\
&= \mathrm{tr}_{c,\gamma} \int \mathrm{d}^4x \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \left[e^{-tD^\dagger(\partial_\mu\to\partial_\mu+ik_\mu)D(\partial_\nu\to\partial_\nu+ik_\nu)} \right] \cdot 1 .
\end{aligned}
\qquad (7)
$$

The unity in eq.(7) points out that the operators here act on unity, so that $\partial_\mu \cdot 1 = 0$. The further calculations are straightforward: the expression in eq.(7) can be expanded in powers of the covariant derivative, integrated with respect to k and the Lorentz indices summed.

3

Since the explicit calculation is given in [7] we present here only some useful formulae and the final result for the determinant up to order $\mathcal{O}(1/m^2)$. The square of the Dirac operator in eq.(7) with all differentiation operators shifted, $\partial_\mu \to \partial_\mu + \mathrm{i}k_\mu$, is given by

$$D^\dagger(\partial_\mu \to \partial_\mu + \mathrm{i}k_\mu)D(\partial_\nu \to \partial_\nu + \mathrm{i}k_\nu) = -\nabla^2 + \frac{\sigma}{2}F - 2\,\mathrm{i}k\nabla + k^2 + m^2 \,, \qquad (8)$$

where we have used that

$$F_{\mu\nu}^a \quad = \mathrm{i}\,[\nabla_\mu, \nabla_\nu]^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f^{abc}A_\mu^b A_\nu^c \qquad (9)$$

$$\Rightarrow \quad -\not\nabla\not\nabla \quad = -\nabla^2 + \frac{\sigma}{2}F \,, \qquad (10)$$

with the notations $\sigma F = \sigma_{\mu\nu}\frac{\lambda^a}{2}F_{\mu\nu}^a$ and $\sigma_{\mu\nu} = \frac{\mathrm{i}}{2}[\gamma_\mu, \gamma_\nu]$ applied. Expanding the exponential function in eq.(7) the functional trace then reads

$$\mathrm{Tr}\left[e^{-tD^\dagger D}\right] = \mathrm{tr}_{c,\gamma}\int \mathrm{d}^4x \int \frac{\mathrm{d}^4k}{(2\pi)^4} e^{-t(k^2+m^2)} \sum_{n=0}^\infty (-1)^n \frac{t^n}{n!}\left(-\nabla^2 + \frac{\sigma}{2}F - 2\,\mathrm{i}k\nabla\right)^n \cdot 1 \quad (11)$$

$$= \frac{1}{4\pi^2}\,\mathrm{tr}_c \int \mathrm{d}^4x\, e^{-tm^2}\left[\frac{1}{t^2} + t^0\left(\frac{1}{6}\nabla_\alpha\nabla_\beta\nabla_\alpha\nabla_\beta - \frac{1}{6}\nabla_\alpha\nabla^2\nabla_\alpha + \frac{1}{4}F_{\alpha\beta}F_{\alpha\beta}\right)\right.$$

$$+ t\left(\frac{1}{180}\nabla^2\nabla^2\nabla^2 - \frac{1}{36}\left(\nabla_\alpha\nabla^2\nabla_\alpha\nabla^2 + \nabla_\alpha\nabla^2\nabla^2\nabla_\alpha + \nabla^2\nabla_\alpha\nabla^2\nabla_\alpha\right)\right.$$

$$+ \frac{1}{45}\left(\nabla_\alpha\nabla_\beta\nabla_\alpha\nabla_\beta\nabla^2 + \nabla_\alpha\nabla_\beta\nabla_\alpha\nabla^2\nabla_\beta + \nabla_\alpha\nabla_\beta\nabla^2\nabla_\alpha\nabla_\beta\right.$$

$$\left. + \nabla_\alpha\nabla^2\nabla_\beta\nabla_\alpha\nabla_\beta + \nabla^2\nabla_\alpha\nabla_\beta\nabla_\alpha\nabla_\beta + \nabla_\alpha\nabla_\beta\nabla^2\nabla_\beta\nabla_\alpha\right)$$

$$- \frac{1}{90}\left(\nabla_\alpha\nabla_\beta\nabla_\alpha\nabla_\gamma\nabla_\beta\nabla_\gamma + \nabla_\alpha\nabla_\beta\nabla_\gamma\nabla_\alpha\nabla_\beta\nabla_\gamma\right.$$

$$\left. + \nabla_\alpha\nabla_\beta\nabla_\gamma\nabla_\alpha\nabla_\gamma\nabla_\beta + \nabla_\alpha\nabla_\beta\nabla_\gamma\nabla_\beta\nabla_\alpha\nabla_\gamma + \nabla_\alpha\nabla_\beta\nabla_\gamma\nabla_\beta\nabla_\gamma\nabla_\alpha\right)$$

$$+ \frac{1}{6}\left(F_{\alpha\beta}F_{\alpha\beta}\nabla^2 + F_{\alpha\beta}\nabla^2 F_{\alpha\beta} + \nabla^2 F_{\alpha\beta}F_{\alpha\beta}\right.$$

$$\left. - \nabla_\gamma F_{\alpha\beta}\nabla_\gamma F_{\alpha\beta} - \nabla_\gamma F_{\alpha\beta}F_{\alpha\beta}\nabla_\gamma - F_{\alpha\beta}\nabla_\gamma F_{\alpha\beta}\nabla_\gamma\right)$$

$$\left.\left. - \frac{\mathrm{i}}{6}F_{\alpha\beta}F_{\gamma\beta}F_{\gamma\alpha}\right) + \mathcal{O}(\nabla^8)\right] \,. \qquad (12)$$

Here we used that all contributions with an odd number of k's vanish whereas all other integrals with respect to k yield

$$\int \frac{\mathrm{d}^4k}{(2\pi)^4} k_{\mu_1}\ldots k_{\mu_{2n}}e^{-t(k^2+m^2)} = \frac{1}{4\pi^2}(2t)^{-(n+2)}e^{-tm^2}\delta_{\mu_1\ldots\mu_{2n}} \,, \qquad (13)$$

with $\delta_{\mu_1\ldots\mu_{2n}}$ denoting all possible contractions:

$$\delta_{\mu_1\ldots\mu_{2n}} = \exp\left[\frac{1}{2}\frac{\partial^2}{\partial\phi_\nu\partial\phi_\nu}\right]\phi_{\mu_1}\ldots\phi_{\mu_{2n}}\Big|_{\phi=0}. \qquad (14)$$

After rearranging the terms into gauge invariants and taking also the part without external gluon fields in eq.(6) into account, the determinant up to order $1/m^2$ can be written as

follows:

$$
\left(\frac{\det D}{\det D_0}\right)_{\zeta-\mathrm{reg}} = \exp\left[\int \mathrm{d}^4x\left(-\frac{1}{48\pi^2}\ln\left(\frac{M^2}{m^2}\right)\mathrm{tr}_c F_{\alpha\beta}F_{\alpha\beta}\right.\right.
$$
$$
-\frac{\mathrm{i}}{720\pi^2}\frac{1}{m^2}\mathrm{tr}_c F_{\alpha\beta}F_{\beta\gamma}F_{\gamma\alpha} + \frac{1}{1440\pi^2}\frac{1}{m^2}\mathrm{tr}_c\left[\nabla_\alpha, F_{\alpha\beta}\right]\left[\nabla_\gamma, F_{\gamma\beta}\right]
$$
$$
-\frac{11}{1440\pi^2}\frac{1}{m^2}\mathrm{tr}_c\left[\nabla_\gamma,\left[\nabla_\alpha, F_{\alpha\beta}\right]\right]F_{\gamma\beta} + \frac{1}{360\pi^2}\frac{1}{m^2}\partial_\alpha\,\mathrm{tr}_c\left[\nabla_\alpha, F_{\gamma\beta}\right]F_{\gamma\beta}
$$
$$
\left.\left.-\frac{1}{384\pi^2}\frac{1}{m^2}\partial^2\,\mathrm{tr}_c F_{\gamma\beta}F_{\gamma\beta}\right) + \mathcal{O}\left(\frac{1}{m^4}\right)\right] \tag{15}
$$
$$
= \exp\left[\int \mathrm{d}^4x\left(-\frac{1}{48\pi^2}\ln\left(\frac{M^2}{m^2}\right)\mathrm{tr}_c F_{\alpha\beta}F_{\alpha\beta}\right.\right.
$$
$$
-\frac{\mathrm{i}}{720\pi^2}\frac{1}{m^2}\mathrm{tr}_c F_{\alpha\beta}F_{\beta\gamma}F_{\gamma\alpha}
$$
$$
\left.\left.+\frac{1}{120\pi^2}\frac{1}{m^2}\mathrm{tr}_c\left[\nabla_\alpha, F_{\alpha\beta}\right]\left[\nabla_\gamma, F_{\gamma\beta}\right]\right) + \mathcal{O}\left(\frac{1}{m^4}\right)\right] . \tag{16}
$$

Note that in the last step partial integration has been used with all total derivatives left out. The effective action $S_{\mathrm{eff,E}} = -\ln\det D$ which can be yielded from (16), rotated back to Minkowski space, corresponds exactly to the result of [6, 7]:

$$
S_{\mathrm{eff,M}} = -\frac{1}{48\pi^2}\int \mathrm{d}^4x\left(\ln\left(\frac{M^2}{m^2}\right)\mathrm{tr}_c F_{\alpha\beta}F^{\alpha\beta}\right.
$$
$$
\left.-\frac{\mathrm{i}}{15\pi^2}\frac{1}{m^2}\mathrm{tr}_c F_{\alpha\beta}F^\beta{}_\gamma F^{\gamma\alpha}\right) + \mathcal{O}\left(\frac{1}{m^4}\right) , \tag{17}
$$

where equation of motion terms, which vanish in pure Yang-Mills theory $\left[\nabla_\alpha, F_{\alpha\beta}\right] = 0$ have been neglected.

2.2 Expansion of heavy quark currents

In order to expand $\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\Gamma|x\rangle$ in eq.(2) in a series of the inverse heavy quark mass we can use eq.(8) to rewrite it as:

$$
\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\Gamma|x\rangle = \mathrm{tr}_{c,\gamma}\int\frac{\mathrm{d}^4k}{(2\pi)^4}e^{-\mathrm{i}kx}\frac{1}{D^\dagger D}D^\dagger\Gamma e^{\mathrm{i}kx}
$$
$$
= \mathrm{tr}_{c,\gamma}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{1}{k^2+m^2}\sum_{n=0}^\infty\left(\frac{\nabla^2 - \frac{\sigma}{2}F + 2\,\mathrm{i}k\nabla}{k^2+m^2}\right)^n\left(\mathrm{i}\,\slashed{\nabla} - \slashed{k} - \mathrm{i}m\right)\Gamma\cdot 1 . \tag{18}
$$

The expansion in eq.(18) is again justified for small gradients of the gluonic fields compared to the heavy quark mass m. Depending on the Lorentz structure Γ some of the integrals might be divergent and need to be regularized, we choose the dimensional regularization, since the integrals in eq.(18) can then be calculated using:

$$
\int\frac{\mathrm{d}^dk}{(2\pi)^d}\frac{k_{\mu_1}k_{\mu_2}\ldots k_{\mu_{2m}}}{(k^2+m^2)^n} = \frac{1}{(4\pi)^{d/2}}\frac{\Gamma\left(n-m-d/2\right)}{\Gamma\left(n\right)2^m}\delta_{\mu_1\ldots\mu_{2m}}\left(\frac{1}{m^2}\right)^{n-m-d/2} . \tag{19}
$$

The number of terms contributing to a given order of $1/m$ is reduced by the fact that terms containing an odd number of γ matrices or an odd number of k's vanish due to the trace over Lorentz-indices and the integration with respect to k. The expansion of eq.(18) then is straightforward. The result of the expansion must be gauge invariant because we expand the gauge invariant operator. In order to obtain explicitely gauge invariant result for heavy quark mass expansion a number of helpful identities based on the Bianchi identity:

$$\left[\nabla_\alpha, F_{\beta\gamma}\right] + \left[\nabla_\gamma, F_{\alpha\beta}\right] + \left[\nabla_\beta, F_{\gamma\alpha}\right] = 0 , \tag{20}$$

can be derived. They will be presented in the following sections.

We want to illustrate some technical details of expanding heavy quark currents on the example of the pseudoscalar density and the divergency of the axial-vector current, which are related to each other by the axial anomaly. Another motivation to show detailed calculation for these cases is that recently confusing results for these cases were reported in the literature [8, 9, 10]. Further we present the result of the expansion of scalar, vector and tensor currents and of $\langle \bar{Q}\nabla_\mu\gamma_\nu Q\rangle$ appearing in the energy-momentum tensor of QCD.

2.2.1 The pseudoscalar density

For $\Gamma = \gamma_5$ the expansion eq.(18) has the form:

$$\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\gamma_5|x\rangle = -\,\mathrm{i}m\,\mathrm{tr}_{c,\gamma}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{1}{k^2+m^2}\sum_{n=0}^{\infty}\left(\frac{\nabla^2-\frac{\sigma}{2}F+2\,\mathrm{i}k\nabla}{k^2+m^2}\right)^n\gamma_5\cdot 1 \tag{21}$$

Collecting all terms which contribute up to $\mathcal{O}(1/m^3)$ one gets:

$$\begin{aligned}
\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\gamma_5|x\rangle &= -\,\mathrm{i}m\,\mathrm{tr}_{c,\gamma}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\Big[\frac{1}{(k^2+m^2)^3}\frac{1}{4}\sigma F\sigma F\gamma_5 \\
&+\frac{1}{(k^2+m^2)^4}\left(\frac{1}{4}\nabla^2\sigma F\sigma F+\frac{1}{4}\sigma F\nabla^2\sigma F+\frac{1}{4}\sigma F\sigma F\nabla^2-\frac{1}{8}\sigma F\sigma F\sigma F\right)\gamma_5 \\
&-\frac{1}{(k^2+m^2)^5}\Big(\sigma F\sigma Fk\nabla k\nabla+\sigma Fk\nabla\sigma Fk\nabla+\sigma Fk\nabla k\nabla\sigma F \\
&\quad+k\nabla\sigma Fk\nabla\sigma F+k\nabla k\nabla\sigma F\sigma F+k\nabla\sigma F\sigma Fk\nabla\Big)\gamma_5\Big]+\mathcal{O}\left(\frac{1}{m^5}\right) \tag{22}\\
&= \frac{\mathrm{i}}{32\pi^2 m}\varepsilon_{\alpha\beta\gamma\delta}\,\mathrm{tr}_c F_{\alpha\beta}F_{\gamma\delta}-\frac{1}{48\pi^2 m^3}\varepsilon_{\alpha\beta\gamma\delta}\,\mathrm{tr}_c F_{\rho\alpha}F_{\rho\beta}F_{\gamma\delta} \\
&+\frac{\mathrm{i}}{192\pi^2 m^3}\varepsilon_{\alpha\beta\gamma\delta}\,\mathrm{tr}_c\Big[F_{\alpha\beta}F_{\gamma\delta}\nabla^2-F_{\alpha\beta}\nabla_\rho F_{\gamma\delta}\nabla_\rho+F_{\alpha\beta}\nabla^2 F_{\gamma\delta} \\
&-\nabla_\rho F_{\alpha\beta}\nabla_\rho F_{\gamma\delta}+\nabla^2 F_{\alpha\beta}F_{\gamma\delta}-\nabla_\rho F_{\alpha\beta}F_{\gamma\delta}\nabla_\rho\Big]+\mathcal{O}\left(\frac{1}{m^5}\right) . \tag{23}
\end{aligned}$$

Here we have used eq.(19) for the integration over k and the following results for Dirac traces:

$$\begin{aligned}
F_{\alpha\beta}F_{\gamma\delta}\mathrm{tr}_\gamma\left[\sigma_{\alpha\beta}\sigma_{\gamma\delta}\gamma_5\right] &= -F_{\alpha\beta}F_{\gamma\delta}\mathrm{tr}_\gamma\left[\gamma_\alpha\gamma_\beta\gamma_\gamma\gamma_\delta\gamma_5\right] \\
&= -4\varepsilon_{\alpha\beta\gamma\delta}F_{\alpha\beta}F_{\gamma\delta} , \tag{24}\\
F_{\alpha\beta}F_{\gamma\delta}F_{\epsilon\varphi}\mathrm{tr}_\gamma\left[\sigma_{\alpha\beta}\sigma_{\gamma\delta}\sigma_{\epsilon\varphi}\gamma_5\right] &= -\,\mathrm{i}F_{\alpha\beta}F_{\gamma\delta}F_{\epsilon\varphi}\mathrm{tr}_\gamma\left[\gamma_\alpha\gamma_\beta\gamma_\gamma\gamma_\delta\gamma_\epsilon\gamma_\varphi\gamma_5\right] \\
&= 16\,\mathrm{i}\varepsilon_{\alpha\beta\gamma\delta}F_{\rho\alpha}F_{\rho\beta}F_{\gamma\delta} \tag{25}
\end{aligned}$$

6

Using the following identities:

$$\left[\nabla_\rho, F_{\alpha\beta}\right]\left[\nabla_\rho, F_{\gamma\delta}\right] = \nabla_\rho F_{\alpha\beta}\nabla_\rho F_{\gamma\delta} + F_{\alpha\beta}\nabla_\rho F_{\gamma\delta}\nabla_\rho - \nabla_\rho F_{\alpha\beta}F_{\gamma\delta}\nabla_\rho - F_{\alpha\beta}\nabla^2 F_{\gamma\delta} \,, \quad (26)$$

$$\left[\nabla_\rho, \left[\nabla_\rho, F_{\alpha\beta}\right]\right]F_{\gamma\delta} = \nabla^2 F_{\alpha\beta}F_{\gamma\delta} + F_{\alpha\beta}\nabla^2 F_{\gamma\delta} - 2\nabla_\rho F_{\alpha\beta}\nabla_\rho F_{\gamma\delta} \,, \quad (27)$$

$$F_{\alpha\beta}\left[\nabla_\rho, \left[\nabla_\rho, F_{\gamma\delta}\right]\right] = F_{\alpha\beta}\nabla^2 F_{\gamma\delta} + F_{\alpha\beta}F_{\gamma\delta}\nabla^2 - 2F_{\alpha\beta}\nabla_\rho F_{\gamma\delta}\nabla_\rho \quad (28)$$

eq.(23) can be written as

$$\begin{aligned}
\operatorname{tr}_{c,\gamma}\langle x|\frac{1}{D}\gamma_5|x\rangle &= \frac{i}{32\pi^2 m}\varepsilon_{\alpha\beta\gamma\delta}\operatorname{tr}_c F_{\alpha\beta}F_{\gamma\delta} \\
&\quad + \frac{i}{192\pi^2 m^3}\varepsilon_{\alpha\beta\gamma\delta}\operatorname{tr}_c\left[2\left[\nabla_\rho, \left[\nabla_\rho, F_{\alpha\beta}\right]\right]F_{\gamma\delta} + \left[\nabla_\rho, F_{\alpha\beta}\right]\left[\nabla_\rho, F_{\gamma\delta}\right]\right. \\
&\qquad\qquad \left. + 4\,i F_{\rho\alpha}F_{\rho\beta}F_{\gamma\delta}\right] + \mathcal{O}\left(\frac{1}{m^5}\right) .
\end{aligned} \quad (29)$$

From the Bianchi identity (20) we obtain the following relations[1]

$$\left[\nabla_\rho, \left[\nabla_\rho, F_{\alpha\beta}\right]\right] = -2\,i\left[F_{\rho\alpha}, F_{\rho\beta}\right] + \left[\nabla_\alpha, \left[\nabla_\rho, F_{\rho\beta}\right]\right] - \left[\nabla_\beta, \left[\nabla_\rho, F_{\rho\alpha}\right]\right], \quad (30)$$

$$\varepsilon_{\alpha\beta\gamma\delta}\operatorname{tr}_c\left[\nabla_\rho, F_{\rho\beta}\right]\left[\nabla_\alpha, F_{\gamma\delta}\right] = 0 \,, \quad (31)$$

so that

$$\begin{aligned}
\varepsilon_{\alpha\beta\gamma\delta}\operatorname{tr}_c\left[\nabla_\rho, \left[\nabla_\rho, F_{\alpha\beta}\right]\right]F_{\gamma\delta} &= -2\,i\varepsilon_{\alpha\beta\gamma\delta}\operatorname{tr}_c\left[F_{\rho\alpha}F_{\rho\beta}F_{\gamma\delta} - F_{\rho\beta}F_{\rho\alpha}F_{\gamma\delta}\right] \\
&\quad + \varepsilon_{\alpha\beta\gamma\delta}\operatorname{tr}_c\left[\left[\nabla_\alpha, \left[\nabla_\rho, F_{\rho\beta}\right]\right]F_{\gamma\delta} - \left[\nabla_\beta, \left[\nabla_\rho, F_{\rho\alpha}\right]\right]F_{\gamma\delta}\right] \\
&= \varepsilon_{\alpha\beta\gamma\delta}\operatorname{tr}_c\left[-4\,i F_{\rho\alpha}F_{\rho\beta}F_{\gamma\delta} + 2\left[\nabla_\alpha, \left[\nabla_\rho, F_{\rho\beta}\right]\right]F_{\gamma\delta}\right] \\
&= \varepsilon_{\alpha\beta\gamma\delta}\left(-4\,i\operatorname{tr}_c F_{\rho\alpha}F_{\rho\beta}F_{\gamma\delta} + 2\partial_\alpha\operatorname{tr}_c\left[\nabla_\rho, F_{\rho\beta}\right]F_{\gamma\delta}\right) .
\end{aligned} \quad (32)$$

On the other hands it yields

$$\begin{aligned}
\varepsilon_{\alpha\beta\gamma\delta}\operatorname{tr}_c\left[\left[\nabla_\rho, \left[\nabla_\rho, F_{\alpha\beta}\right]\right]F_{\gamma\delta} + \left[\nabla_\rho, F_{\alpha\beta}\right]\left[\nabla_\rho, F_{\gamma\delta}\right]\right] &= \varepsilon_{\alpha\beta\gamma\delta}\operatorname{tr}_c\left[\nabla_\rho, \left[\nabla_\rho, F_{\alpha\beta}\right]F_{\gamma\delta}\right] \\
&= \varepsilon_{\alpha\beta\gamma\delta}\partial_\rho\operatorname{tr}_c\left[\nabla_\rho, F_{\alpha\beta}\right]F_{\gamma\delta} \\
&= \varepsilon_{\alpha\beta\gamma\delta}\partial_\rho\operatorname{tr}_c\left[\left[\nabla_\rho, F_{\alpha\beta}F_{\gamma\delta}\right] - F_{\alpha\beta}\left[\nabla_\rho, F_{\gamma\delta}\right]\right] \\
&= \frac{1}{2}\varepsilon_{\alpha\beta\gamma\delta}\partial_\rho\operatorname{tr}_c\left[\nabla_\rho, F_{\alpha\beta}F_{\gamma\delta}\right] \\
&= \frac{1}{2}\varepsilon_{\alpha\beta\gamma\delta}\partial^2\operatorname{tr}_c F_{\alpha\beta}F_{\gamma\delta} .
\end{aligned} \quad (33)$$

So we finally end up with

$$\begin{aligned}
\operatorname{tr}_{c,\gamma}\langle x|\frac{1}{D}\gamma_5|x\rangle &= \frac{i}{32\pi^2 m}\varepsilon_{\alpha\beta\gamma\delta}\operatorname{tr}_c F_{\alpha\beta}F_{\gamma\delta} \\
&\quad + \frac{i}{384\pi^2 m^3}\varepsilon_{\alpha\beta\gamma\delta}\partial^2\operatorname{tr}_c F_{\alpha\beta}F_{\gamma\delta} + \frac{i}{96\pi^2 m^3}\varepsilon_{\alpha\beta\gamma\delta}\partial_\alpha\operatorname{tr}_c\left[\nabla_\rho, F_{\rho\beta}\right]F_{\gamma\delta} + \mathcal{O}\left(\frac{1}{m^5}\right) \\
&= \frac{i}{16\pi^2 m}\operatorname{tr}_c F_{\alpha\beta}\tilde{F}_{\alpha\beta} \\
&\quad + \frac{i}{192\pi^2 m^3}\partial^2\operatorname{tr}_c F_{\alpha\beta}\tilde{F}_{\alpha\beta} + \frac{i}{48\pi^2 m^3}\partial_\alpha\operatorname{tr}_c\left[\nabla_\rho, F_{\rho\beta}\right]\tilde{F}_{\alpha\beta} + \mathcal{O}\left(\frac{1}{m^4}\right) ,
\end{aligned} \quad (34)$$

---

[1] In the calculation of [9] the factor of 2 was missing in the first identity, which led to the wrong result.

7

where we have introduced the common notation $\tilde{F}_{\alpha\beta} = \frac{1}{2}\varepsilon_{\alpha\beta\gamma\delta}F_{\gamma\delta}$.

2.2.2 The divergency of the axial vector current

Instead of expanding the axial vector current $j^5_\mu(x) = Q^\dagger(x)\gamma_\mu\gamma_5 Q(x)$ in the way outlined, we can use that the divergence of the axial vector current is given by

$$\partial_\mu j^5_\mu = 2mQ^\dagger\gamma_5 Q - \frac{\mathrm{i}}{16\pi^2}F^a_{\mu\nu}\tilde{F}^a_{\mu\nu}\,, \tag{35}$$

where the first term contains the axial current and the second is the axial anomaly term which arises due to quantum effects. The expansion of the divergence of the axial vector current in terms of the inverse of the heavy quark mass is therefore reduced to the expansion of the axial current, which we have already performed before.

Further the axial anomaly equation (35) has some general properties, which can be used to check our result for the axial current:

First the rhs of eq. (35) vanishes in the limit of infinite quark mass. This can be understood by the fact that the regulator mass cancels the physical mass in the infinite mass limit because of the different sign in the definition of the regulator. Therefore we expect the order $\mathcal{O}(1/m)$ term in the expectation value of the axial current multiplied by $2m$ exactly to cancel the anomaly term. Indeed equation (34) gives:

$$2m\operatorname{tr}_{c,\gamma}\langle x|\frac{1}{D}\gamma_5|x\rangle^{\mathcal{O}(1/m)} = \frac{\mathrm{i}}{8\pi^2}\operatorname{tr}_c F_{\mu\nu}\tilde{F}_{\mu\nu}\,. \tag{36}$$

Second if one thinks of the expectation value of the axial vector current as a local large m expansion in the background of gluon fields

$$\langle j^5_\mu(x)\rangle = \sum_n \frac{1}{m^n}X^{(n)}_{\mu 5}(x) \tag{37}$$

then due to equation (35) the expectation value of the axial current in the large m expansion is

$$2m\operatorname{tr}_{c,\gamma}\langle x|\frac{1}{D}\gamma_5|x\rangle = \sum_n \frac{1}{m^n}\partial_\mu X^{(n)}_{\mu 5}(x)\,. \tag{38}$$

This means that terms appearing in the expansion of the axial current must be of the form of a total derivative. The order $\mathcal{O}(1/m^3)$ term in equation (34) exactly obeys this form

$$2m\operatorname{tr}_{c,\gamma}\langle x|\frac{1}{D}\gamma_5|x\rangle^{\mathcal{O}(1/m^3)} = \frac{\mathrm{i}}{96\pi^2 m^2}\partial_\mu R_\mu\,, \tag{39}$$

$$R_\mu = \partial_\mu \operatorname{tr}_c F_{\alpha\beta}\tilde{F}_{\alpha\beta} + 4\operatorname{tr}_c\Big[\nabla_\alpha, F_{\alpha\nu}\Big]\tilde{F}_{\mu\nu}\,. \tag{40}$$

The term $f^{abc}F^a_{\mu\nu}\tilde{F}^b_{\nu\alpha}F^c_{\alpha\mu}$ appearing falsely in the expansion of the axial current in [8, 9] cannot be represented as a total derivative of a local expression[2] and therefore violates the general argument given above.

---

[2] A straightforward calculation for the instanton field shows that $\int \mathrm{d}^4 x f^{abc}F^a_{\mu\nu}\tilde{F}^b_{\nu\alpha}F^c_{\alpha\mu} \neq 0$. But for dimensional reasons this nonvanishing contribution can be excluded from being generated by a surface term if the instanton field is taken in the regular gauge. Therefore the integrand cannot be a total derivative.

The expectation value for the divergence of the axial vector current in the background of gluon fields finally reads up to order $\mathcal{O}(1/m^4)$

$$\langle \partial_\mu j_\mu^5(x) \rangle = \frac{\mathrm{i}}{96\pi^2 m^2} \partial_\mu \left(\partial_\mu \, \mathrm{tr}_c F_{\alpha\beta} \tilde{F}_{\alpha\beta} + 4 \, \mathrm{tr}_c \left[\nabla_\alpha, F_{\alpha\nu} \right] \tilde{F}_{\mu\nu} \right) + \mathcal{O}\left(\frac{1}{m^4} \right) . \tag{41}$$

2.2.3 The scalar current

Following the steps outlined in the introduction to this section the expansion of a scalar current in series of the inverse heavy quark mass yields up to order $\mathcal{O}(1/m^3)$:

$$
\begin{aligned}
\mathrm{tr}_{c,\gamma} \langle x | \frac{1}{D} | x \rangle &= \mathrm{tr}_{c,\gamma} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{k^2 + m^2} \sum_{n=0}^{\infty} \left(\frac{\nabla^2 - \frac{\sigma}{2} F + 2 \, \mathrm{i} k \nabla}{k^2 + m^2} \right)^n (\mathrm{i} \nabla\!\!\!/ - k\!\!\!/ - \mathrm{i} m) \cdot 1 \\
&= \frac{-\mathrm{i}}{(4\pi)^{\frac{d}{2}}} \left(\frac{1}{m^2} \right)^{1 - \frac{d}{2}} m \Gamma \left(1 - \frac{d}{2} \right) d \, \mathrm{tr}_c 1 \\
&\quad - \frac{\mathrm{i}}{24\pi^2 m} \, \mathrm{tr}_c F_{\alpha\beta} F_{\alpha\beta} \\
&\quad + \frac{1}{360\pi^2 m^3} \, \mathrm{tr}_c F_{\alpha\beta} F_{\alpha\gamma} F_{\beta\gamma} - \frac{7\mathrm{i}}{2880\pi^2 m^3} \partial^2 \, \mathrm{tr}_c F_{\alpha\beta} F_{\alpha\beta} \\
&\quad - \frac{\mathrm{i}}{720\pi^2 m^3} \left(11 \, \mathrm{tr}_c \left[\nabla_\alpha, \left[\nabla_\beta, F_{\beta\gamma} \right] \right] F_{\alpha\gamma} - \mathrm{tr}_c \left[\nabla_\alpha, F_{\alpha\beta} \right] \left[\nabla_\gamma, F_{\gamma\beta} \right] \right) .
\end{aligned}
\tag{42}
$$

The infinite constant term can be cancelled by substracting the expectation value of the scalar current for vanishing gluonic background fields. Our result eq. (42) coincides with that obtained in ref. [11] if we neglect the total derivative terms which were ignored in ref. [11].

Actually the result eq. (42) with the total derivative terms neglected (and hence that of ref. [11]) can be easily obtained from the expansion of the determinant of the Dirac operator (15), since

$$
\begin{aligned}
\int \mathrm{d}^4 x \, \mathrm{tr}_{c,\gamma} \langle x | \frac{1}{D} | x \rangle &= \frac{\mathrm{d}}{\mathrm{d}m} \left(- \mathrm{i} \ln \left(\det D \right) \right) \\
&= \frac{\mathrm{d}}{\mathrm{d}m} \left(- \mathrm{i} \, \mathrm{Tr} \ln D \right) .
\end{aligned}
\tag{43}
$$

Our expansion of the scalar current (42) is in agreement with the result for the determinat in equation (15).

2.2.4 The vector current

The heavy quark expansion of the vector current up to order $1/m^3$ gives exactly zero

$$\mathrm{tr}_{c,\gamma} \langle x | \frac{1}{D} \gamma_\mu | x \rangle = 0 + \mathcal{O}\left(\frac{1}{m^4} \right) . \tag{44}$$

This result can be easily anticipated from the fact that the vector current is $C-$parity odd. This implies that the first operator contributing to heavy quark mass expansion should contain at least three gluon fields, additionaly the vector current conservation requires that this operator has the following structure ∇G^3. From counting of dimensions we see that such operator can contribute only at $1/m^4$ order.

9

2.2.5 The tensor current

For the color singlet tensor current we find that the first non-vanishing order of the expansion is $\mathcal{O}\left(1/m^3\right)$, yielding

$$
\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\sigma_{\mu\nu}|x\rangle = \frac{\mathrm{i}}{24\pi^2}\frac{1}{m^3}
$$
$$
\times \mathrm{tr}_c\left[F_{\alpha\beta}F_{\alpha\beta}F_{\mu\nu} + F_{\alpha\nu}F_{\beta\mu}F_{\alpha\beta} - F_{\alpha\mu}F_{\beta\nu}F_{\alpha\beta}\right] + \mathcal{O}\left(\frac{1}{m^5}\right). \quad (45)
$$

We note that the rhs of the above equation vanishes in the case of $SU(2)$ gauge group. Actually one can show that the lhs of eq. (45) is identically zero in the case of $SU(2)$ gauge group. Therefore the fact that rhs of eq. (45) vanishes for $SU(2)$ gauge group is a powerful check of our calculations.

In order to prove that lhs of eq. (45) is zero in the case of SU(2) gauge group we use the following transformation:

$$
G = C\tau^2\,,
$$

where C is charge conjugation matrix in Dirac spinor space and τ^2 is color $SU(2)$ matrix. Under this transformation we have:

$$
\begin{aligned}
G\tau^a G^{-1} &= -\tau^{aT} \\
G\sigma_{\mu\nu}G^{-1} &= -\sigma_{\mu\nu}^T \\
GDG^{-1} &= D^T
\end{aligned}
$$

where ^{T} is the transposition operation. The lhs. of eq. (45) should be zero, since

$$
\begin{aligned}
\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\sigma_{\mu\nu}|x\rangle &= \mathrm{tr}_{c,\gamma}\langle x|G\frac{1}{D}\sigma_{\mu\nu}G^{-1}|x\rangle \\
&= \mathrm{tr}_{c,\gamma}\langle x|\left(\frac{1}{D}\right)^T\left(-\sigma_{\mu\nu}^T\right)|x\rangle \\
&= -\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\sigma_{\mu\nu}|x\rangle\,. \quad (46)
\end{aligned}
$$

Nullification of the heavy quark mass expansion of the tensor current for $SU(2)$ gauge group implies that lhs. of eq.(45) is zero if it is computed in the field of single instanton.

2.2.6 Expansion of $\langle \bar{Q}\nabla_\mu\gamma_\nu Q\rangle$.

The energy-momentum tensor of QCD can be written in Minkowski-space as

$$
T^{\mu\nu} = -g^{\mu\nu}\mathcal{L}_{\mathrm{QCD}} - F^{\mu\alpha}F_\alpha^\nu + \frac{\mathrm{i}}{2}\bar{\psi}\overset{\leftrightarrow}{\nabla}{}^{(\mu}\gamma^{\nu)}\psi\,, \quad (47)
$$

where $(\mu\nu)$ denotes the symmetrization of the indices. The large m expansion of the (not symmetrized) last term in eq. (47) yields in Euclidean space

$$
\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\nabla_\mu\gamma_\nu|x\rangle = \frac{-2\,\mathrm{i}}{(4\pi)^{\frac{d}{2}}}\left(\frac{1}{m^2}\right)^{-\frac{d}{2}}\Gamma\left(-\frac{d}{2}\right)\delta_{\mu\nu}\,\mathrm{tr}_c 1
$$

$$+\frac{i}{(4\pi)^{\frac{d}{2}}}\left(\frac{1}{m^2}\right)^{2-\frac{d}{2}}\Gamma\left(2-\frac{d}{2}\right)\left(-\frac{1}{3}\delta_{\mu\nu}\,\mathrm{tr}_c F_{\alpha\beta}F_{\alpha\beta}+\frac{4}{3}\,\mathrm{tr}_c F_{\alpha\nu}F_{\alpha\mu}\right)$$

$$+\frac{1}{720\pi^2}\frac{1}{m^2}\delta_{\mu\nu}\,\mathrm{tr}_c F_{\alpha\beta}F_{\beta\gamma}F_{\gamma\alpha}-\frac{7\,i}{5760\pi^2}\frac{1}{m^2}\delta_{\mu\nu}\partial^2\,\mathrm{tr}_c F_{\alpha\beta}F_{\alpha\beta}$$

$$-\frac{i}{1440\pi^2}\frac{1}{m^2}\delta_{\mu\nu}\left(11\,\mathrm{tr}_c\big[\nabla_\alpha,\big[\nabla_\beta,F_{\beta\gamma}\big]\big]F_{\alpha\gamma}-\mathrm{tr}_c\big[\nabla_\alpha,F_{\alpha\beta}\big]\big[\nabla_\gamma,F_{\gamma\beta}\big]\right)$$

$$+\frac{1}{2880\pi^2}\frac{1}{m^2}\Big(-4\,\mathrm{tr}_c F_{\alpha\nu}F_{\beta\mu}F_{\alpha\beta}-4\,\mathrm{tr}_c F_{\alpha\mu}F_{\beta\nu}F_{\alpha\beta}$$

$$-30\,i\,\mathrm{tr}_c\big[\nabla_\alpha,F_{\mu\nu}\big]\big[\nabla_\beta,F_{\alpha\beta}\big]$$

$$+74\,i\,\mathrm{tr}_c\big[\nabla_\alpha,\big[\nabla_\beta,F_{\beta\nu}\big]\big]F_{\alpha\mu}+14\,i\,\mathrm{tr}_c\big[\nabla_\alpha,\big[\nabla_\beta,F_{\beta\mu}\big]\big]F_{\alpha\nu}$$

$$-26\,i\,\mathrm{tr}_c\big[\nabla_\nu,\big[\nabla_\alpha,F_{\alpha\beta}\big]\big]F_{\beta\mu}-26\,i\,\mathrm{tr}_c\big[\nabla_\mu,\big[\nabla_\alpha,F_{\alpha\beta}\big]\big]F_{\beta\nu}$$

$$+22\,i\partial^2\,\mathrm{tr}_c F_{\beta\nu}F_{\beta\mu}-3\,i\partial_\mu\partial_\nu\,\mathrm{tr}_c F_{\alpha\beta}F_{\alpha\beta}$$

$$-26\,i\,\mathrm{tr}_c\big[\nabla_\mu,F_{\alpha\nu}\big]\big[\nabla_\beta,F_{\alpha\beta}\big]-26\,i\,\mathrm{tr}_c\big[\nabla_\nu,F_{\alpha\mu}\big]\big[\nabla_\beta,F_{\alpha\beta}\big]$$

$$+40\,i\,\mathrm{tr}_c\big[\nabla_\alpha,F_{\alpha\nu}\big]\big[\nabla_\beta,F_{\beta\mu}\big]+4\,i\,\mathrm{tr}_c\big[\nabla_\nu,F_{\alpha\beta}\big]\big[\nabla_\mu,F_{\alpha\beta}\big]\Big)+\mathcal{O}\left(\frac{1}{m^4}\right)\quad(48)$$

and

$$\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\nabla_\mu\gamma_\nu|x\rangle^{\mathcal{O}(m^0,m^{-2})}=\;\mathrm{tr}_{c,\gamma}\langle x|\nabla_\mu\frac{1}{D}\gamma_\nu|x\rangle^{\mathcal{O}(m^0,m^{-2})}.\qquad(49)$$

The Lorentz trace of eq. (48) can be compared with the expansion of the scalar current in eq. (42) since

$$\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\nabla_\nu\gamma_\nu|x\rangle=-m\,\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}|x\rangle.\qquad(50)$$

For the trace of the rhs of eq. (48) we find

$$\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\nabla_\nu\gamma_\nu|x\rangle^{\mathcal{O}(m^0,m^{-2})}=\;\frac{i}{24\pi^2}\,\mathrm{tr}_c F_{\alpha\beta}F_{\alpha\beta}$$

$$-\frac{1}{360\pi^2 m^2}\,\mathrm{tr}_c F_{\alpha\beta}F_{\alpha\gamma}F_{\beta\gamma}+\frac{7\,i}{2880\pi^2 m^2}\partial^2\,\mathrm{tr}_c F_{\alpha\beta}F_{\alpha\beta}$$

$$+\frac{i}{720\pi^2 m^2}\left(11\,\mathrm{tr}_c\big[\nabla_\alpha,\big[\nabla_\beta,F_{\beta\gamma}\big]\big]F_{\alpha\gamma}-\mathrm{tr}_c\big[\nabla_\alpha,F_{\alpha\beta}\big]\big[\nabla_\gamma,F_{\gamma\beta}\big]\right),\qquad(51)$$

which exactly coincides with the rhs of eq. (42) multiplied by $(-m)$. Eq. (49) further agrees with the expansion of the vector current beeing zero up to $\mathcal{O}(1/m^3)$ since

$$\mathrm{tr}_{c,\gamma}\langle x|\nabla_\mu\frac{1}{D}\gamma_\nu|x\rangle-\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\nabla_\mu\gamma_\nu|x\rangle=\partial_\mu\,\mathrm{tr}_{c,\gamma}\langle x|\frac{1}{D}\gamma_\nu|x\rangle.\qquad(52)$$

3 Intrinsic heavy quarks in light hadrons

In light hadron processes heavy quarks may give contributions only through virtual effects which are supresed by the mass of the heavy quarks. Especially for the charm quark, whose mass $m_c \approx 1.4$ GeV is not too large, virtual processes nevertheless may give not negligible contributions. In this section we discuss the applications of heavy quark mass expansion obtained in the previous sections.

In the following sections we use the "perturbative" normalization for the gluon field strength $G^a_{\mu\nu} = F^a_{\mu\nu}/g$ and rotate all expressions to Minkowsky space, see the appendix.

3.1 Intrinsic charm in η and η'

For the decay of the B-meson into η' and K-mesons in [8] a mechanism with virtual charm quarks was suggested. In this approach the Cabbibo favored process $b \to \bar{c}cs$ is followed by the conversion of the $\bar{c}c$ pair directly into η'. Its contribution to the decay amplitude is therefore direct depending on the "intrinsic charm" component of the η'-meson which is usually characterized by the matrix element

$$\langle 0|\bar{c}\gamma_\mu\gamma_5 c|\eta'(q)\rangle = i f^{(c)}_{\eta'} q_\mu \,. \tag{53}$$

Using the heavy mass expansion of the divergence of the axial vector current (41), the constant $f^{(c)}_{\eta'}$ can be expressed up to the order $1/m_c^2$ by

$$f^{(c)}_{\eta'} = -\frac{1}{12 m_c^2} \langle 0|\frac{\alpha_s}{4\pi} G^a_{\mu\nu}\tilde{G}^{\mu\nu,a}|\eta'\rangle \,. \tag{54}$$

Here we neglected the term proportional to $\left[\nabla_\alpha, G^\alpha_\nu\right]$ in (41) which vanishes in pure Yang-Mills theory. We now can estimate the value of the constant $f^{(c)}_{\eta'}$:

$$f^{(c)}_{\eta'} \approx -2\,\text{MeV}\,, \tag{55}$$

where we have used

$$\langle 0|\frac{\alpha_s}{4\pi} G^a_{\mu\nu}\tilde{G}^{\mu\nu,a}|\eta'\rangle = 0.056\,\text{GeV}^3\,, \tag{56}$$

obtained in [12].

In QCD the omitted term can be related to the matrix element

$$\frac{\alpha_s}{4\pi}\langle 0|g \sum_{q=u,d,s} \bar{q}\gamma_\nu\tilde{G}^\nu_\mu q|\eta'\rangle \tag{57}$$

using the equation of motion. A rough order of magnitude estimate for the contribution of the omitted term (57) to $f^{(c)}_{\eta'}$ using the results of [13] indicates an deviation at the level of 0.3 MeV to the value (55). A more careful analysis of the omitted matrix element (57) can be done by using the instanton methods developed in [14] which already have been applied by [13] to calculations of higher twist corrections to deep-inelastic scattering.

Our estimated value for $f^{(c)}_{\eta'}$ is consistent with the phenomenological analysis in [15] where the authors derivied the bound $-65\,\text{MeV} \le f^{(c)}_{\eta'} \le 15\,\text{MeV}$ from the analysis of $\gamma\eta'$ transition form factors. From the analysis of (η, η', η_c)-mixing in [16] the small value $f^{(c)}_{\eta'} = -(6.3 \pm 0.6)$ MeV was derived, taking into account off-shellness effects in the $\bar{c}c$ component of η' also, the value $|f^{(c)}_{\eta'}| \approx 2.4$ MeV was found in [17]. Further our value for $f^{(c)}_{\eta'}$ is in agreement with the phenomenological bound $|f^{(c)}_{\eta'}| < 12$ MeV, obtained in [18], and corresponds to the result $f^{(c)}_{\eta'} \approx -2.3$ MeV presented in [19]. In Ref. [19] the

12

divergence of the axial vector current was computed using the triangle graph for the axial anomaly with massive fermions, neglecting possible $1/m_c^2$ contributions like

$$f^{abc}G_{\mu\nu}^a \tilde{G}_{\nu\alpha}^b G_{\alpha\mu}^c \tag{58}$$

from higher order diagrams. Indeed our calculation shows that such "truly nonabelean" operators do not contribute to the order $1/m_c^2$ and our result (54) therefore is exactly given by the first term of the expansion in $1/m_c^2$ of the triangle graph [20].

The small value (55) for $f_{\eta'}^{(c)}$ implies that the $b \to \bar{c}cs$ mechanism does not play a major role in the $B \to K\eta'$ decay mode.

Bigger values of $f_{\eta'}^{(c)}$ due to the operator (58) in the expansion of the axial current up to order $1/m_c^3$ have been given in [8], where $f_{\eta'}^{(c)} \approx (50 - 180)$ MeV and in [9] with $f_{\eta'}^{(c)} \approx -(12.3 - 18.4)$ MeV. These results have been used by a number of authors for the analysis of the charm content in noncharmed hadrons (see e.g [21, 10, 22, 23]), but since the operator (58) violates general properties of the axial anomaly and it also does not appear in explicit calculations (see section 2.2.2), results relying on [8, 9] should be reconsidered.

Analogously we can immediately estimate the constant $f_{\eta}^{(c)}$ characterizing the intrinsic charm contribution to the η-meson. Using

$$\langle 0|\frac{\alpha_s}{4\pi}G_{\mu\nu}^a \tilde{G}^{\mu\nu,a}|\eta\rangle = 0.020\,\text{GeV}^3 \,, \tag{59}$$

obtained in [12] we find

$$f_{\eta}^{(c)} \approx -0.7\,\text{MeV} \,. \tag{60}$$

Since in the case of the η meson the contribution of the omitted term (57) can be of the same order as $f_{\eta}^{(c)}$ itself, the estimate (60) must be considered as a poor.

3.2 Intrinsic charm contribution to the proton spin

Another application of our result for the heavy quark mass expansion of the divergency of axial vector current (41) has been given in [20]. In this paper the authors have shown that the intrinsic charm contribution to the first moment of the spin structure function $g_1(x, Q^2)$ of the nucleon is small contrary to the result of [21, 22, 10].

In ref. [20] it was proven that the forward matrix element of the axial current in the leading order of heavy quark mass expansion can be computed as:

$$\langle N(p,\lambda)|\bar{c}\gamma_\mu\gamma_5 c(0)|N(p,S)\rangle = \frac{\alpha_s}{48\pi m_c^2}\langle N(p,S)|R_\mu(0)|N(p,S)\rangle \,. \tag{61}$$

Here the current $R_\mu(0)$ is given by eq. (40). Note that the first term in R_μ does not contribute to the forward matrix element because of its gradient form, while the contribution of the second one is rewritten, by making use of the equation of motion, as matrix element of the operator

$$\begin{aligned}
\langle N(p,S)|\bar{c}\gamma_\mu\gamma_5 c(0)|N(p,S)\rangle &= \frac{\alpha_s}{12\pi m_c^2}\langle N(p,S)|g\sum_{f=u,d,s}\bar{\psi}_f\gamma_\nu\tilde{G}_\mu{}^\nu\psi_f|N(p,S)\rangle \\
&\equiv \frac{\alpha_s}{12\pi m_c^2}2m_N^3 S_\mu f_S^{(2)}, \tag{62}
\end{aligned}$$

13

The parameter $f_S^{(2)}$ was determined before in calculations of the power corrections to the first moment of the singlet part of g_1. QCD-sum rule calculations gave $f_S^{(2)} = 0.09$ [24], estimates using the renormalon approach led to $f_S^{(2)} = \pm 0.02$ [25] and calculations in the instanton model of the QCD vacuum give a result very close to that of QCD sum rule [13].

Inserting these numbers we get finally for the charm axial constant of the nucleon the estimate

$$g_A^{(c)} = -\frac{\alpha_s}{12\pi} f_S^{(2)} \frac{m_N^2}{m_c^2} \approx -5 \cdot 10^{-4} \tag{63}$$

with probably a 100 percent uncertainty. Note that this contribution is of non-perturbative origin (therefore we call it intrinsic), so that it is sensitive to large distances, as soon as the factorization scale is of order m_c.

3.3 Intrinsic charm contribution to the nucleon tensor charge

Using the results of section 2.2.5 we can estimate the intrinsic charm contribution to the tensor charge of the nucleon. The tensor charge of the nucleon is defined as:

$$\langle N(p,S)|\bar{c}\sigma_{\mu\nu}\gamma_5 c(0)|N(p,S)\rangle = 2\,\mathrm{i}\,g_T^{(c)}(p_\mu S_\nu - p_\nu S_\mu)\,. \tag{64}$$

Using the result of the section 2.2.5 and the identity

$$\sigma_{\mu\nu}\gamma_5 = -\frac{\mathrm{i}}{2}\varepsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta}\,,$$

we obtain for the charm contribution to the nucleon tensor charge the following result:

$$g_T^{(c)} = \frac{1}{96\pi^2}\frac{1}{m_c^3 m_N^2}$$

$$\times\varepsilon^{\lambda\rho\mu\nu}p_\lambda S_\rho\langle N(p,S)|\,\mathrm{tr}_c g_s^3\Big[G_{\alpha\beta}G^{\alpha\beta}G_{\mu\nu} + 2\,G_{\alpha\nu}G_{\beta\mu}G^{\alpha\beta}\Big]|N(p,S)\rangle + \mathcal{O}\left(\frac{1}{m_c^5}\right) \tag{65}$$

The matrix element in the rhs of above equation can be rouhgly estimated in the instanton vacuum using method of [14]. As discussed in section 2.2.5 the gluonic operator in rhs of eq. (65) is identically zero on one instanton. Therefore the first nontrivial contribution one can get from instanton–anti-instanton pair. If we compare the expression for the charm contribution to the nucleon charge with that for axial charge we see that the charm contribution to the tensor charge is suppressed by additional power of m_N/m_c and one power of instanton packing fraction $(\pi^2\bar{\rho}^4/\bar{R}^4)$, however the tensor charge is enchanced by one power of $\alpha_s(m_c)$ [3]. This allows us to make a rough estimate for the charm contribution of the tensor charge:

$$g_T^{(c)} \sim \frac{m_N}{\alpha_s(m_c)\,m_c}\frac{(N_c-2)\,\pi^2\,\bar{\rho}^4}{N_c\,\bar{R}^4}\,g_A^{(c)} \sim 10^{-4}\,. \tag{66}$$

_____

[3] Let us note that the expansion parameter in the heavy quark mass expansion is $g_s G/m_c$, because the non-perturbative gluon field strength $G \sim 1/g_s$ (_cf._ instanton field). Therefore $g_s(\mu)$ accompanied by gluon field strength is not counted as suppression.

Factors of N_c are written in a way to reproduce the large N_c behaviour of the matrix element and to account for the fact that the operator in rhs of eq. (65) is identically zero at $N_c = 2$.

3.4 Intrinsic charm contribution to the nucleon momentum

The charm contribution to the nucleon momentum can be defined as:

$$M_2^{(c)}(\mu^2) = \int_0^1 dx_B \, x_B \left[c(x_B) + \bar{c}(x_B) \right] = \frac{\mathrm{i}}{2(P \cdot n)^2} \langle N(P) | \bar{c} \slashed{n} (n \cdot \nabla) c(0) | N(P) \rangle, \quad (67)$$

where $c(x_B)$ is the charm parton distribution normalized at the scale μ, which is assumed to be $\mu \approx m_c$. The light cone vector n is arbitrary non-collinear to nucleon momentum P.

Now we can use the result of eq. (48) in order to estimate the charm contribution to the nucleon momentum carried by intrinsic charm quarks.

$$M_2^{(c)}(\mu) = \frac{\mathrm{i}}{2(P \cdot n)^2} \left[\frac{\mathrm{i}\,\alpha_s(\mu)}{4\pi} \frac{1}{\left(2 - \frac{d}{2}\right)} \frac{4}{3} \langle N(P) | n^\mu n^\nu \, \mathrm{tr}_c G^\alpha{}_\nu G_{\alpha\mu} | N(P) \rangle \right.$$

$$\left. + \frac{1}{120\pi^2} \frac{g_s^3(\mu)}{m_c^2} \langle N(P) | n^\mu n^\nu \, \mathrm{tr}_c G_{\alpha\nu} G_{\beta\mu} G^{\alpha\beta} | N(P) \rangle \right] + \mathcal{O}\left(\frac{1}{m_c^4}\right). \quad (68)$$

In derivation of this expression we neglected terms which are proportional to $[\nabla^\alpha, G_{\alpha\beta}]$ which are suppressed by one power of $g_s(\mu)^2$. The first term in eq. (68) is divergent[4] and actually is related to the mixing of quark and gluon operators. We can rewrite eq. (68) as follows:

$$M_2^{(c)}(\mu) = \frac{4}{3} \frac{\alpha_s(\mu)}{4\pi} \frac{1}{\left(2 - \frac{d}{2}\right)} \, M_2^{(G)}(\mu)$$

$$+ \quad \frac{\mathrm{i}}{2(P \cdot n)^2} \frac{1}{120\pi^2} \frac{g_s^3(\mu)}{m_c^2} \langle N(P) | n^\mu n^\nu \, \mathrm{tr}_c G_{\alpha\nu} G_{\beta\mu} G^{\alpha\beta} | N(P) \rangle + \mathcal{O}\left(\frac{1}{m_c^4}\right). \quad (69)$$

Here the first term, which is proportional to the momentum fraction carried by gluons $M_2^{(G)}(\mu)$ accounts for extrinsic charm. Note that the coefficient in front of this term is exactly the leading anomalous dimension $\gamma_{qG} = 4/3$ which accounts for mixing quark and gluon twist-2 operators under QCD evolution. The intrinsic charm contribution is given by the second term, so that we have estimates:

$$M_2^{(c),\,\mathrm{intrinsic}}(\mu) = \frac{\mathrm{i}}{2(P \cdot n)^2} \frac{1}{120\pi^2} \frac{g_s^3(\mu)}{m_c^2} \langle N(P) | n^\mu n^\nu \, \mathrm{tr}_c G_{\alpha\nu} G_{\beta\mu} G^{\alpha\beta} | N(P) \rangle + \mathcal{O}\left(\frac{1}{m_c^4}\right). (70)$$

We see that the momentum fraction carried by intrinsic charm in the nucleon is related to the value of nucleon matrix element:

$$\langle N(P) | n^\mu n^\nu \, \mathrm{i} g_s(\mu)^3 \, \mathrm{tr}_c G_{\alpha\nu} G_{\beta\mu} G^{\alpha\beta} | N(P) \rangle.$$

---

[4]We show only the most singular term

One can easily see that this matrix element in the theory of instanton vacuum [14] is zero in one-instanton approximation, the same as matrix element

$$\langle N(P)|n^\mu n^\nu g_s(\mu)^2 \operatorname{tr}_c G^\alpha{}_\nu G_{\alpha\mu}|N(P)\rangle \,.$$

Keeping in mind that for instanton field $G \sim 1/g_s$ we can write:

$$\frac{\langle N(P)|n^\mu n^\nu \; ig_s(\mu)^3 \operatorname{tr}_c G_{\alpha\nu} G_{\beta\mu} G^{\alpha\beta}|N(P)\rangle}{\langle N(P)|n^\mu n^\nu g_s(\mu)^2 \operatorname{tr}_c G^\alpha_\nu G_{\alpha\mu}|N(P)\rangle} = \Lambda^2 \,, \tag{71}$$

where Λ is parameter of the dimension of mass whose value can be obtained using various nonperturbative methods in QCD: lattice calculation, QCD sum rule, theory of instanton vacuum. Generically we expect that this mass parameter is of order of typical strong interaction scale $\Lambda \sim 1$ GeV. Now we can rewrite eq. (70) in terms of this parameter and momentum fraction carried by gluons in the nucleon at scale $\mu \approx m_c$ as:

$$M_2^{(c), \, \text{intrinsic}}(\mu) = \frac{\alpha_s(\mu)}{30\pi} \frac{\Lambda^2}{m_c^2} M_2^G(\mu) + \mathcal{O}\left(\frac{1}{m_c^4}\right) \,. \tag{72}$$

If we assume that $\Lambda^2 =$few GeV^2 than we get the estimate for the charm contribution to the nucleon momentum:

$$M_2^{(c), \, \text{intrinsic}}(\mu) = \text{ few} \times 10^{-3} \,. \tag{73}$$

We see that the heavy quark mass expansion of local currents allows us to reduce the problem of estimate of intrinsic charm content of the nucleon to the calculation of the ratio (71). The latter ratio can be computed using various methods of nonperturbative QCD, probably the most promising would be a calculation of this ratio in lattice QCD.

Recent analysis of refs [26, 27] gives for $M_2^{(c), \, \text{intrinsic}}$ values at the level of fraction of percent what is in agrrement with our estimate (73).

Let us note that, since we performed the heavy quark mass expansion of heavy quark part of energy momentum tensor not neglecting total derivatives, one can compute also its non-forward nucleon matrix element. From the non-forward matrix element of energy momentum one can obtain the total angular momentum carried by intrinsic heavy quarks in the nucleon using Ji's sum rules [28]. The corresponding estimates we shall report elsewhere.

4   Conclusions

In this paper we have computed the heavy quark mass expansion of various local heavy quark currents. The details of the technique are illustrated on the example of heavy quark mass expansion of the pseudoscalar density $\bar{Q}\gamma_5 Q$. This operator plays an important role in problems related to intrinsic charm contribution to the proton spin and to intrinsic charm content of η, η' mesons.

We corrected the mistakes in refs. [8, 9] for heavy quark mass expansion of the operator $\bar{Q}\gamma_5 Q$. In these papers large intrinsic charm contribution to the proton spin and to intrinsic charm content of η, η' mesons was obtained due to contribution of the operator

$f^{abc}G^a_{\mu\nu}\tilde{G}^b_{\nu\alpha}G^c_{\alpha\mu}$ which appeared in heavy quark mass expansion of the operator $\bar{Q}\gamma_5 Q$ presented in refs. [8, 9]. We showed that coefficient in front of this operator is identically zero (the result which actually follows from general properties of the axial anomaly [4]), so that the physical effects based on presence of the above operator discussed in refs. [8, 9, 21, 10, 22, 23] are absent.

For the first time we presented the full results[5] for heavy quark mass expansion of the operators $\bar{Q}Q$ (to the order $1/m^3$), $\bar{Q}\gamma_5 Q$ (to the order $1/m^3$), $\partial^\mu \bar{Q}\gamma_\mu\gamma_5 Q$ (to the order $1/m^2$), $\bar{Q}\gamma_\mu Q$ (to the order $1/m^3$), $\bar{Q}\sigma_{\mu\nu}Q$ (to the order $1/m^3$), and $\bar{Q}\gamma_\mu\nabla_\nu Q$ (to the order $1/m^2$).

The results obtained for heavy quark mass expansion allowed us to estimate the intrinsic charm content of η', η mesons as well the charm contribution to the proton spin, nucleon tensor charge and to the fraction of nucleon momentum carried by intrinsic charm. In the case of charm content of η', η mesons and intrinsic charm contributions to the proton spin we reduce the calculations of these quantities to matrix elements which are already known either phenomenologically or were computed previously. In other cases, like intrinsic charm contribution to the nucleon tensor charge and to energy momentum tensor, the problem is reduced to matrix elements of gluon operators which can be estimated using various nonperturbative methods in QCD: lattice calculation, QCD sum rule, theory of instanton vacuum.

We made here rough order of magnitude estimate of matrix elements of gluon operators appearing in heavy quark mass expansion of tensor current and of energy momentum tensor using instanton model of QCD vacuum. More quantitative estimates will be given elsewhere. The predictions for intrinsic charm contribution to various observables are summarized in Table 1.

quantity	our estimate
$f^{(c)}_{\eta'}$	-2 MeV
$f^{(c)}_{\eta}$	-0.7 MeV
$g^{(c)}_A$	$-5 \cdot 10^{-4}$
$g^{(c)}_T$	$\sim 10^{-4}$
$M^{(c),\,\text{intrinsic}}_2$	$\sim 10^{-3}$

Table 1: *Results for intrinsic charm contribution to various observables.*

5   Appendix

For the euclidization we use the follwing conventions:

---

[5]Not neglecting total derivatives and terms proportional to $[\nabla^\mu, G_{\mu\nu}]$

$$
\begin{aligned}
\mathrm{i}x_{\mathrm{M}}^{0} &= x_{4,\,\mathrm{E}}\,, & x_{\mathrm{M}}^{k} &= x_{k,\mathrm{E}} & &\rightarrow \mathrm{d}^4 x_{\mathrm{M}} = -\mathrm{i}\,\mathrm{d}^4 x_{\mathrm{E}}\,, \\
\partial_{\mathrm{M}}^{0} &= \mathrm{i}\partial_{4,\mathrm{E}}\,, & \partial_{\mathrm{M}}^{k} &= -\partial_{k,\mathrm{E}}\,, \\
A_{\mathrm{M}}^{0} &= \mathrm{i}A_{4,\mathrm{E}}\,, & A_{\mathrm{M}}^{k} &= -A_{k,\mathrm{E}}\,.
\end{aligned}
\tag{74}
$$

The covariant derivative therefore reads in Minkowski and in Euclidean space-time:

$$
\nabla_{\mathrm{M}}^{\mu} = \left(\partial^{\mu} - \mathrm{i}A^{\mu}(x)\right)_{\mathrm{M}}\,, \tag{75}
$$

$$
\nabla_{\mu,\mathrm{E}} = \left(\partial_{\mu} - \mathrm{i}A_{\mu}(x)\right)_{\mathrm{E}}\,. \tag{76}
$$

The field strength, defined as

$$
F_{\mu\nu}^{a} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} + f^{abc}A_{\mu}^{b}A_{\nu}^{c} = \mathrm{i}\left[\nabla_{\mu},\nabla_{\nu}\right] \tag{77}
$$

transforms as

$$
F_{ij,\mathrm{M}} = F_{ij,\mathrm{E}}\,, \quad F_{0j,\mathrm{M}} = \mathrm{i}F_{4j,\mathrm{E}}\,. \tag{78}
$$

For the Dirac matrices we choose the conventions:

$$
\gamma_{\mathrm{M}}^{0} = \gamma_{4,\mathrm{E}}\,, \quad \gamma_{\mathrm{M}}^{k} = \mathrm{i}\gamma_{k,\mathrm{E}}\,, \tag{79}
$$

and γ_5 is defined within this paper as:

$$
\gamma_{5,\mathrm{M}} = \gamma_{\mathrm{M}}^{5} = -\mathrm{i}\left(\gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3}\right)_{\mathrm{M}} = \left(\gamma_1\gamma_2\gamma_3\gamma_4\right)_{\mathrm{E}} = \gamma_{5,\mathrm{E}}\,. \tag{80}
$$

With

$$
\varepsilon_{\mathrm{M}}^{0123} = -\varepsilon_{0123,\mathrm{M}} = +1 = \varepsilon_{1234,\mathrm{E}}\,, \tag{81}
$$

it yields

$$
\mathrm{tr}_{\gamma}\left[\gamma_5\gamma_{\alpha}\gamma_{\beta}\gamma_{\gamma}\gamma_{\delta}\right]_{\mathrm{E}} = 4\varepsilon_{\alpha\beta\gamma\delta,\mathrm{E}}\,. \tag{82}
$$

The fermionic fields transform as

$$
\psi_{\mathrm{M}} = \psi_{\mathrm{E}}\,, \quad \bar{\psi}_{\mathrm{M}} = -\mathrm{i}\psi_{\mathrm{E}}^{\dagger}\,, \tag{83}
$$

so the Dirac operator in Euclidean space-time reads:

$$
D = \mathrm{i}\,\slashed{\nabla} + \mathrm{i}m\,. \tag{84}
$$

In section 2.1 we have used the following transformation properties for the effective action and the appearing operators:

$$
S_{\mathrm{eff,M}} = \mathrm{i}S_{\mathrm{eff,E}}\,, \tag{85}
$$

$$
\left(F_{\alpha\beta}F^{\alpha\beta}\right)_{\mathrm{M}} = \left(F_{\alpha\beta}F_{\alpha\beta}\right)_{\mathrm{E}}\,, \quad \left(F_{\alpha\beta}F^{\beta}{}_{\gamma}F^{\gamma\alpha}\right)_{\mathrm{M}} = -\left(F_{\alpha\beta}F_{\beta\gamma}F_{\gamma\alpha}\right)_{\mathrm{E}}\,. \tag{86}
$$

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